Spin Liquid States at the vicinity of metal-insulator transition

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We study in this letter quantum spin liquid states (QSLs) at the vicinity of metal-insulator transition. Assuming that the low energy excitations in the QSLs are labeled by "spinon" occupation numbers with the same Fermi surface structure as in the corresponding metal (Fermi-liquid) side, we propose a phenomenological Landau-like low energy theory for the QSLs and show that the usual $U(1)$ QSLs is a representative member of this class of spin liquids. Based on our effective low energy theory, an alternative picture to the Brinkman-Rice picture of Mott metal-insulator transition is proposed. The charge, spin and thermal responses of QSLs are discussed under such a phenomenology.

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Quantum spin liquid states (QSLs) in dimensions $d > 1$ has been a long sought dream in condensed matter physics which has not been confirmed in realistic materials until rather recently.[1] These states are electronic Mott insulators that are not magnetically ordered down to the lowest temperature due to strong quantum mechanical fluctuations of spins and/or frustrated interaction. Various exotic properties have been proposed to exist in QSLs. For instance, charge neutral and spin-$\frac{1}{2}$ mobile objects, spinons, were proposed to emerge in such electronic states at low temperature accompanied by different kinds of (emerging) gauge fields. The spinons may be gapped or gapless and may obey either boson or fermion statistics. These new particles and gauge fields which characterize low energy behaviors of the system do not appear in the parent Hamiltonian and "emerge" as a result of strong correlation.

In the past few years, several experimental candidates for QSLs have been discovered that support the existence of gapless fermionic spinon excitations. The best studied example is a family of organic compounds $\kappa-(ET)_2Cu_2(CN)_3$ (ET)[2] and $Pd(dmit)_2(EtMe_3Sb)$ (dmit salts)[3]. Both materials are Mott insulators in proximity to the metal-insulator transition because they become superconductor (ET) or metal (dmit) under modest pressure. Despite the large magnetic exchange $J \approx 250$ K observed in these systems, there is no experimental indication of long range magnetic ordering down to temperature $\sim 30\ mK$. Linear temperature dependence of the specific heat and Pauli-like spin susceptibility were found in both materials at low temperature suggesting that the low energy excitations are spin-1/2 fermions with a Fermi surface.[4, 5] This Fermi liquid-like behavior is further supported by their Wilson ratios which are close to one. The thermal conductivity experiments on the ET salts found a large contribution to κ beside phonons with κ/T much reduced below 0.3 K,[6] while κ/T approaches to a constant down to the lowest temperature in dmit salts.[7] All these experimental observations point to the scenario that the low lying excitations in these Mott insulators are mobile fermionic particles (spinons) that form a Fermi surface like their parent electrons. Besides ET and dmit-salt, the Kagome compound $ZnCu_3(OH)_6Cl_2$, the three dimensional hyper-Kagome material $Na_4Ir_3O_8$ and the newly discovered triangular compound $Ba_3CuSb_2O_9$ are also considered to be candidates for QSLs with gapless excitations.[8–10]

Several experiments were proposed to probe mobile spinons. For example, giant magnetoresistance like experiment was designed to measure mobile spinons through oscillatory coupling between two ferromagnets via a quantum spin liquid spacer.[11] The thermal Hall effect in insulating quantum magnets was proposed as an example of thermal transport of spinons, where different responses were used to distinguish between magnon- and spinon- transports.[12] The spinon life time and mass can be measured through sound attenuation experiment.[13] Despite all these proposals, a generic method to compare theoretical prediction of QSLs to experimental data is still missing at the phenomenological level.

The purpose of this letter is to build a general phenomenological theory for spin liquids with (large) Fermi surfaces. We observe that like Fermi liquid states, the low energy excitations in these QSLs are assumed to be labeled by the same occupation numbers as free fermions, the difference between Fermi liquids and QSLs is that the excitations in Fermi liquid are quasi-particles that carry both charges and spins, whereas the excitations in QSLs carry only spins. In particular, DC charge transport exists in Fermi liquid states, but is absent in QSLs (insulators).

The fact that the low energy excitations in these QSLs are labeled by the same occupation numbers as free fermions suggests that the excitation energy $\Delta E = E - E_G$ for these states are also given by a Landau-type

expression[14, 15]

$$\Delta E=\sum_{\mathrm{p}\sigma}\xi_{\mathrm{p}}\delta n_{\mathrm{p}\sigma}+\frac{1}{2}\sum_{\mathrm{pp}'\sigma\sigma'}f^{\sigma\sigma'}_{\mathrm{pp}'}\delta n_{\mathrm{p}\sigma}\delta n_{\mathrm{p}'\sigma'}+O(\delta n^3), \tag{1}$$

where $\xi_{\mathrm{p}}=\frac{\mathrm{p}^2}{2m^*}-\mu$ is the (single) spinon energy measured from the chemical potential μ, m^* is the spinon effective mass and σ and σ' are spin indices. $\delta n_{\ \mathrm{p}\sigma}=n_{\mathrm{p}\sigma}-n^0_{\mathrm{p}\sigma}$ measures the departure of the spinon distribution function from the ground state distribution $n^0_{\mathrm{p}}=\theta(-\xi_{\mathrm{p}})$. $f^{\sigma\sigma'}_{\mathrm{pp}'}$ is the interaction energy between excited spinons. A spherical, rotational invariant Fermi surface is assumed here for simplicity. In this case we may write $f^{\sigma\sigma'}_{\mathrm{pp}'}$ in terms of spin symmetric and spin antisymmetric components $f^{\sigma\sigma'}_{\mathrm{pp}'}=f^{s}_{\mathrm{pp}'}\delta_{\sigma\sigma'}+f^{a}_{\mathrm{pp}'}\sigma\sigma'$. For isotropic systems, $f^{s(a)}_{\mathrm{pp}'}$ depends only on the angle θ between p and p$'$ and we can expand $f^{s(a)}_{\mathrm{pp}'}=\sum_{l=0}^{\infty}f^{s(a)}_{l}P_l(\cos\theta)$ at 3D and $f^{s(a)}_{\mathrm{pp}'}=\sum_{l=0}^{\infty}f^{s(a)}_{l}\cos(l\theta)$ at 2D, where P_l's are Legendre polynomials. The Landau parameters, defined by

$$F^{s(a)}_l=N(0)f^{s(a)}_l,$$

provide a dimensionless measures of the strengths of the interactions between spinons on the Fermi surface, where $N(0)$ is the Fermi surface density of states. The low temperature properties of the QSLs are completely determined by the spinon mass m^* and the interaction $f^{\sigma\sigma'}_{\mathrm{pp}'}$ (or $F^{s(a)}_l$) as in Fermi liquid theory.

Notice that the energy functional ΔE for our QSLs is so far identical to that for Fermi liquids. We propose and shall demonstrate in the following that the QSLs distinguish themselves from Fermi liquids by having a strong constraint on the Landau parameters F^s_1.

We start with the observation that the charge current $\mathbf{J}$ carried by quasi-particles in Fermi liquid theory (and in QSLs) is given by

$$\mathbf{J}=\frac{m}{m^*}(1+\frac{F^s_1}{d})\mathbf{J}^{(0)}, \tag{2a}$$

where $\mathbf{J}^{(0)}$ is the charge current carried by the corresponding non-interacting fermions and d is the dimension. For translational invariant systems, the charge current carried by quasi-particles is not renormalized and $\frac{m^*}{m}=1+\frac{F^s_1}{d}$[15]. However, this is in general not valid for electrons in crystals where Galilean invariance is lost. In this case $\frac{m^*}{m}\neq 1+\frac{F^s_1}{d}$ and the charge current carries by quasi-particles is renormalized by quasi-particle interaction. On the other hand, the thermal current $\mathbf{J}_Q$ is only renormalized by the effective mass in Fermi liquid theory,

$$\mathbf{J}_Q=\frac{m}{m^*}\mathbf{J}^{(0)}_Q, \tag{2b}$$

where $\mathbf{J}^{(0)}_Q$ is the corresponding thermal current carried by non-interacting electrons. (See supplementary materials for details.) Thus, in the special case $1+F^s_1/d\to 0$ while $\frac{m^*}{m}$ remaining finite, $\mathbf{J}\to 0$ and $\mathbf{J}_Q\neq 0$ suggesting that the electronic system is in a special state where spin-1/2 quasi-particles do not carry charge due to interaction but they still carry entropy (i.e. electric insulating but thermal conducting). This is exactly what we expect for spinons in QSLs. We note that it is crucial that F^s_1 is independent of $\frac{m^*}{m}$ for this mechanism to work.

To put our argument in a more quantitative framework we study the electromagnetic responses of a Fermi liquid system with $1+F^s_1/d\to 0$. The charge and (transverse) current response functions are given by the standard Fermi liquid forms[14, 16, 17]

$$\chi_d(\mathrm{q},\omega)=\frac{\chi_{0d}(\mathrm{q},\omega)}{1-\left(F^s_0+\frac{F^s_1(\mathrm{q},\omega)}{d+F^s_1(\mathrm{q},\omega)}\frac{\omega^2}{q^2}\right)\frac{\chi_{0d}(\mathrm{q},\omega)}{N(0)}}, \tag{3}$$

and

$$\chi_t(\mathrm{q},\omega)=\frac{\chi_{0t}(\mathrm{q},\omega)}{1-\frac{F^s_1(\mathrm{q},\omega)}{d+F^s_1(\mathrm{q},\omega)}\frac{\chi_{0t}(\mathrm{q},\omega)}{N(0)}}, \tag{4}$$

where $\chi_{0d}(\mathrm{q},\omega)$ and $\chi_{0t}(\mathrm{q},\omega)$ are the density-density and (transverse) current-current response functions for a Fermi gas with effective mass m^* but without Landau interactions, respectively. The longitudinal current-current response function χ_l is related to χ_d through $\chi_d(\mathrm{q},\omega)=(q^2/\omega^2)\chi_l(\mathrm{q},\omega)$ and the ac conductivity $\sigma_{l(t)}$ is related to $\chi_{l(t)}$ by $\sigma_{l(t)}(\mathrm{q},\omega)=e^2\chi_{l(t)}(\mathrm{q},\omega)/i\omega$, where $\mathrm{q}=|\vec{q}|$. In the singular limit $1+F^s_1/d\to 0$, it is clear that higher order q,ω-dependent terms should be included in the Landau interaction to obtain finite results. Expanding at small q and ω, we obtain

$$\frac{1+F^s_1(\mathrm{q},\omega)/d}{N(0)}\sim\alpha-\beta\omega^2+\gamma_t q_t^2+\gamma_l q_l^2, \tag{5}$$

where $q_t\sim\nabla\times$ and $q_l\sim\nabla$ are associated the transverse (curl) and longitudinal (gradient) parts of the small $\vec{q}$ expansion. $\alpha=0$ in the QSLs. Putting this into the charge response function χ_d, we find that to ensure that the system is in an incompressible (insulator) state, we must have $\gamma_l=0$. The other possibility $F^s_0\to\infty$ implies complete vanishing of charge responses in the insulating state.

With this parametrization we obtain for the ac conductivity at small ω,

$$\sigma(\omega)=\frac{\omega\sigma_0(\omega)}{\omega-i\sigma_0(\omega)/\beta e^2}, \tag{6}$$

where $\sigma_0(\omega)=e^2\chi_{0t}(0,\omega)/(i\omega)=e^2\chi_{0l}(0,\omega)/(i\omega)$. The last equality is valid as long as F^s_0 is finite. Eq. (6) was first obtained in the $U(1)$ gauge theory approach to spin liquid in a slightly different form[18] and predicts power law conductivity $\mathrm{Re}[\sigma(\omega)]\sim\omega^{3.33}(\omega^2)$ (at 2D) at frequency regime $\omega>(<)(1/\tau_0,k_BT/\hbar)$, where τ_0 is

the elastic scattering time[18]. The dielectric function is given at small q, ω by

$$\varepsilon(\mathrm{q},\omega)=1-\frac{4\pi e^2}{q^2}\chi_d(\mathrm{q},\omega)\sim 1+4\pi\beta e^2+O(q^2), \quad (7)$$

also in agreement with the result obtained in U(1) gauge theory[18].

The U(1) spin liquid is actually a member of the QSLs described by our phenomenology. To show this we start with a Landau Fermi liquid with interaction parameters F_0^s and $F_1^s(q,\omega)$ only. The long-wavelength and low dynamics of the Fermi liquid is described by an effective Lagrangian

$$L_{\text{eff}}=\sum_{\mathrm{k},\sigma}\left[c^\dagger_{\mathrm{k}\sigma}(i\frac{\partial}{\partial t}-\xi_{\mathrm{k}})c_{\mathrm{k}\sigma}-H'(c^\dagger,c)\right], \quad (8)$$

where $c^\dagger_{\mathrm{k}\sigma}(c_{\mathrm{k}\sigma})$ are spin-σ fermion creation (annihilation) operators with momentum k, and

$$H'(c^\dagger,c)=\frac{1}{2N(0)}\sum_q\left[\frac{F_1^s}{v_F^2}\mathbf{j}(q)\cdot\mathbf{j}(-q)+F_0^s n(q)n(-q)\right] \quad (9)$$

describes the current-current and density-density interactions between quasi-particles[17], where $q=(\mathrm{q},\omega)$ and $v_F=\hbar k_F/m^*$ is the Fermi velocity.

The current- and density- interactions can be decoupled by introducing fictitious gauge potentials $\mathbf{a}$ and φ (Hubbard-Stratonovich transformation) with

$$H'(c^\dagger,c)\to\sum_q\left[\mathbf{j}\cdot\mathbf{a}+n\varphi-\frac{1}{2}\left(\frac{n}{m^*}\frac{d}{F_1^s}\mathbf{a}^2+\frac{N(0)}{F_0^s}\varphi^2\right)\right], \quad (10)$$

where n is fermion density. We have used the equality $d(n/m^*)=N(0)v_F^2$ in writing down Eq. (10).

The Lagrangian (8) and (10) can be rewritten in the standard form of $U(1)$ gauge theory by noting that the fermion current is given in this representation by

$$\mathbf{j}=\frac{-i}{2m^*}\sum_\sigma\left[\psi^\dagger_\sigma\nabla\psi_\sigma-(\nabla\psi^\dagger_\sigma)\psi_\sigma\right]-\frac{n}{m^*}\mathbf{a},$$

where $\psi_\sigma(\mathrm{r})=\int e^{-i\mathrm{k}\cdot\mathrm{r}}c_{\mathrm{k}\sigma}$ in the Fourier transform of $c_{\mathrm{k}\sigma}$. The Lagrangian can be written as

$$L=\sum_\sigma\int d^d\mathrm{r}\left[\psi^\dagger_\sigma(i\frac{\partial}{\partial t}-\varphi)\psi_\sigma-H(\psi^\dagger_\sigma,\psi_\sigma)\right]+L(\varphi,\mathbf{a}), \quad (11a)$$

where

$$H(\psi^\dagger_\sigma,\psi_\sigma)=\frac{1}{2m^*}|(\nabla-i\mathbf{a})\psi_\sigma|^2 \quad (11b)$$

and

$$L(\varphi,\mathbf{a})=\frac{1}{2}\int d^d\mathrm{r}\left[\frac{n}{m^*}(1+\frac{d}{F_1^s})\mathbf{a}^2+\frac{N(0)}{F_0^s}\varphi^2\right]. \quad (11c)$$

Notice how the $\frac{n}{2m^*}\mathbf{a}^2$ term in $L(\varphi,\mathbf{a})$ arises from the introduction of diamagnetic term in $H(\psi^\dagger_\sigma,\psi_\sigma)$.

Using Eq. (5), we find that in the small q limit, the transverse part of $L(\varphi,\mathbf{a})$ is given in the spin liquid state by

$$L_t(\varphi,\mathbf{a})=-\frac{n}{2m^*}\int d^d\mathrm{r}\left[\beta(\frac{\partial\mathbf{a}}{\partial t})^2-\gamma_t(\nabla\times\mathbf{a})^2\right]. \quad (12)$$

The longitudinal part of the gauge potential φ is screened as long as $F_0^s\neq 0$. Lagrangian (11) together with (12) is the standard Lagrangian used to describe $U(1)$ QSLs. It is interesting to note that a nonzero $1+\frac{F_1^s(0,0)}{d}$ leads to a mass term for the gauge field $\mathbf{a}$, in agreement with slave-boson/rotor approaches where a metallic state appears with condensation of bosons/rotors[16, 19].

Our picture of QSLs has several immediate experimental consequences. For example, the specific heat ratio $\gamma^*=C_V/T=m^*/mC_V^{(0)}$ is predicted to be finite at the QSLs where $C_V^{(0)}$ is the specific heat for the corresponding non-interacting electron liquid. The magnetic susceptibility given by

$$\chi_P=\frac{m^*}{m}\frac{1}{1+F_0^a}\chi_P^{(0)},$$

is also non-singular in the QSLs with Wilson ratio $R=\frac{4\pi^2k_B^2\chi_P}{3(g\mu_B)^2\gamma^*}\sim(1+F_0^a)^{-1}$ generally of order $O(1)$. The transport properties in the QSLs can be computed using the Landau transport equation. We expect that the transport life-times will be dominated by scattering in the current-current channel which is the most singular scattering channel in the limit $1+\frac{F_1^s}{d}\to 0$. The effect of scattering in this channel should be equivalent to results obtained from $U(1)$ gauge theory. The thermal conductivity κ computed from Landau transport equation is given by

$$\frac{\kappa}{T}\sim\max\left[\frac{\hbar}{k_B^2}\left(\frac{k_BT}{\epsilon_F}\right)^{\frac{(4-d)}{3}},\frac{d}{\gamma^*v_F^2}\frac{1}{\tau_0}\right]^{-1},$$

where $1/\tau_0$ is the elastic (impurity) scattering rate, $\epsilon_F=p_F^2/2m^*$ is the spinon Fermi energy and v_F is the spinon Fermi velocity. The same result is obtained in U(1) gauge theory at 2D.[20] (See supplementary material for details.)

The close relation between Fermi liquid and spin-liquid states suggests that the (zero temperature) metal-insulator transition between the two states is characterized by the change of Landau parameter $1+F_1^s(0,0)/d\to 0^+$ across the transition. The nature of metal-insulator transition within the Fermi liquid framework was first addressed by Brinkman and Rice[21] where they proposed that a metal-insulator (Mott) transition is indicated by diverging effective mass $\frac{m^*}{m}\to\infty$ and inverse compressibility $\kappa\to 0$ at the Mott transition point, with correspondingly a vanishing quasi-particle renormalization

weight $Z \sim \frac{m}{m^*} \to 0$. The diverging effective mass and vanishing quasi-particle weight suggest that the Fermi liquid state is destroyed at the Mott transition, and the Mott insulator state is distinct from the Fermi liquid state at the metal side. Here we propose an alternative picture where the Fermi surface is not destroyed but the quasiparticles are converted into spinons at the Mott transition. In particular, the effective mass m^*/m may not diverge at the metal-insulator transition although $Z \to 0$ in this picture. A schematic phase diagram for the Mott (metal-QSLs) transition is presented in Fig. (1) where we imagine a Hubbard type Hamiltonian with hopping t and on-site Coulomb repulsion U. The system is driven to a Mott insulator state at zero temperature at $U = U_c$, where $1 + F_1^s(U > U_c)/d = 0$. We caution that in general a finite ($T \neq 0$) region exists around the Mott transition point where the physics is dominated by critical fluctuations and our phenomenological theory is not applicable. We note that an alternative phenomenology for Mott transition from a semi-microscopic starting point[22] has qualitatively similar conclusion as our present work. The relation between the two works is not clear at present.

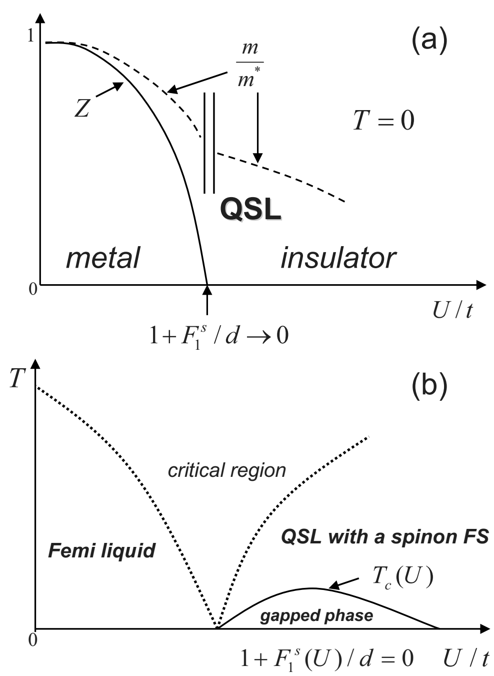

FIG. 1: (a) Schematic zero temperature phase diagram for Mott transition. U is the Hubbard interaction strength and t is the hopping integral. The electron quasiparticle weight and quasi-particle charge current $\sim 1 + F_1^s/d$ vanishes at the critical point while the effective mass remains finite. (b) Schematic phase diagram showing finite temperature crossovers and possible instability toward gapped phases at lower temperature. There exists a (finite temperature) critical region around U_c where our phenomenological theory is not applicable.

We end with a comment about the Pomeranchuk instability. Experienced researchers in Fermi liquid theory will recognize that the point $1 + F_1^s/d = 0$ is in fact a critical point in Fermi liquid theory. The Fermi surface is unstable with respect to deformation when $1+F_1^s/d < 0$. The stability of the $1 + F_1^s/d = 0$ point is required in QSLs where quasi-particles (spinons) become chargeless. The resulting QSLs we obtain here are marginally stable because of large critical fluctuations. Similar result was obtained in $U(1)$ gauge theory, where it was found that a fermionic spin liquid state with large Fermi surface is marginally stable toward $U(1)$ gauge fluctuations. The Pomeranchuk criticality is an alternative way to express this result.

The presence of Pomeranchuk criticality suggests that QSLs with large Fermi surfaces are in general rather susceptible to formation of other more stable QSLs at lower temperature, like the Z_2 QSLs or valence bond solid (VBS) states that gap out part or the whole Fermi surface. The resulting phase diagram at the vicinity of Mott transition thus has the generic feature shown in Fig.(1b), where the system is driven into a gapped QSL at low temperature $T < T_c(U)$ at the insulating side. The nature of the low temperature QSLs depends on the microscopic details of the system and cannot be determined from our phenomenology. Our theory is applicable at $T > T_c(U)$, when the spin liquid is still in the large-Fermi surface phase.

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Supplementary Materials

RENORMALIZED CURRENTS

In this supplementary material we derive the renormalized currents in Fermi liquid theory ((Eq.(2)) in the main text). The local equilibrium quasi-particle occupation numbers and their fluctuations have to be considered carefully. The excitation energy of an additional quasi-particle with momentum p is given by

$$\tilde{\epsilon}_{\mathrm{p}} = \epsilon_{\mathrm{p}} + \sum_{\mathrm{p}'} f^s_{\mathrm{pp}'} \delta n_{\mathrm{p}'},$$

where $\epsilon_{\mathrm{p}} = \frac{p^2}{2m^*}$. The corresponding local equilibrium occupation number is $\tilde{n}^0_{\mathrm{p}} \equiv n_F\left(\tilde{\epsilon}_{\mathrm{p}} - \mu\right)$, and the departure from local equilibrium reads

$$\begin{aligned} \delta\tilde{n}_{\mathrm{p}} &= n_{\mathrm{p}} - \tilde{n}^0_{\mathrm{p}} \\ &= \delta n_{\mathrm{p}} - \frac{\partial n^0}{\partial \epsilon_{\mathrm{p}}} \sum_{\mathrm{p}'} f^s_{\mathrm{pp}'} \delta n_{\mathrm{p}'}, \end{aligned}$$

where $\delta n_{\mathrm{p}} = n_{\mathrm{p}} - n^0_{\mathrm{p}}$. The charge current J carried by quasiparticles is related to the particle density by the conservation law,

$$\frac{\partial \rho}{\partial t} + \nabla_{\mathrm{r}} \cdot \mathbf{J} = 0.$$

The density fluctuation $\delta\rho\left(\mathrm{r}, t\right)$ should be expressed in terms of the sum of $\delta\tilde{n}_{\mathrm{p}}\left(\mathrm{r}, t\right)$ (i.e. fluctuation away from local equilibirum),

$$\delta\rho\left(\mathrm{r}, t\right) = \sum_{\mathrm{p}} \delta\tilde{n}_{\mathrm{p}}\left(\mathrm{r}, t\right)$$

and

$$\frac{\partial}{\partial t}\delta\rho + \nabla_{\mathrm{r}} \cdot \sum_{\mathrm{p}} \delta\tilde{n}_{\mathrm{p}} \mathbf{v}_{\mathrm{p}} = 0.$$

Therefore,

$$\mathbf{J} = \sum_{\mathrm{p}} \delta\tilde{n}_{\mathrm{p}} \mathbf{v}_{\mathrm{p}} = \sum_{\mathrm{p}} \delta n_{\mathrm{p}} \mathbf{j}_{\mathrm{p}},$$

where

$$\mathbf{j}_{\mathrm{p}} = \mathbf{v}_{\mathrm{p}} - \sum_{\mathrm{p}'} f^s_{\mathrm{pp}'} \frac{\partial n^0}{\partial \epsilon_{\mathrm{p}'}} \mathbf{v}_{\mathrm{p}'}.$$

Using the relation

$$\begin{aligned} \sum_{\mathrm{p}'} \frac{\partial n^0}{\partial \epsilon_{\mathrm{p}'}} f_{\mathrm{pp}'} \mathbf{v}_{\mathrm{p}'} &= \sum_{\mathrm{p}'} \frac{\partial n^0}{\partial \epsilon_{\mathrm{p}'}} N_F^{-1} \sum_l F^s_l P_l\left(\cos\theta\right) \mathbf{v}_{\mathrm{p}'} \\ &= \frac{1}{d} F^s_1 \mathbf{v}_{\mathrm{p}} \int d\epsilon' \frac{\partial n^0}{\partial \epsilon'} = \frac{1}{d} F^s_1 \mathbf{v}_{\mathrm{p}}, \end{aligned}$$

where $N_F = N\left(0\right)$, we find that the renormalized charge current is

$$\mathbf{J} = \frac{m}{m^*}(1 + \frac{1}{d}F^s_1)\mathbf{J}^{(0)},$$

when $\mathbf{J}^0$ is the electric current in the absence of interaction. Notice that $P_l\left(\cos\theta\right)$ is replaced by $\cos\left(l\theta\right)$ at 2D.

Similarly the thermal (energy) current $\mathbf{J}_Q$ is given by

$$\begin{aligned} \mathbf{J}_Q &= \sum_{\mathrm{p}} \delta\tilde{n}_{\mathrm{p}} \left(\epsilon_{\mathrm{p}} - \mu\right) \mathbf{v}_{\mathrm{p}} \\ &= \sum_{\mathrm{p}} \left(\epsilon_{\mathrm{p}} - \mu\right) \mathbf{v}_{\mathrm{p}} (\delta n_{\mathrm{p}} - \sum_{\mathrm{p}'} \frac{\partial n^0}{\partial \epsilon_{\mathrm{p}}} f_{\mathrm{pp}'} \delta n_{\mathrm{p}'}) \\ &= \sum_{\mathrm{p}} \delta n_{\mathrm{p}} [\left(\epsilon_{\mathrm{p}} - \mu\right) \mathbf{v}_{\mathrm{p}} - \sum_{\mathrm{p}'} \frac{\partial n^0}{\partial \epsilon_{\mathrm{p}'}} f_{\mathrm{pp}'} \left(\epsilon_{\mathrm{p}'} - \mu\right) \mathbf{v}_{\mathrm{p}'}]. \end{aligned}$$

Notice that

$$\begin{aligned} &\sum_{\mathrm{p}'} \frac{\partial n^0}{\partial \epsilon_{\mathrm{p}'}} f_{\mathrm{pp}'} \left(\epsilon_{\mathrm{p}'} - \mu\right) \mathbf{v}_{\mathrm{p}'} \\ &= \sum_{\mathrm{p}'} \frac{\partial n^0}{\partial \epsilon_{\mathrm{p}'}} N_F^{-1} \sum_l F^s_l P_l\left(\cos\theta\right) \left(\epsilon_{\mathrm{p}'} - \mu\right) \mathbf{v}_{\mathrm{p}'} \\ &= \frac{1}{d} F^s_1 \mathbf{v}_{\mathrm{p}} \int d\epsilon' \frac{\partial n^0}{\partial \epsilon'} \left(\epsilon' - \mu\right) = 0 \end{aligned}$$

to leading order. Therefore the renormalized energy current is given by

$$\mathbf{J}_Q = \frac{m}{m^*} \mathbf{J}^{(0)}_Q.$$

We observe that the thermal current is not renormalized by the factor $\left(1 + \frac{1}{d}F^s_1\right)$.

LANDAU TRANSPORT EQUATION

In the Landau transport equation which is essentially a Boltzmann equation, the transition probability $W(1,2;3,4)$ for a two quasi-particles scattering process in an isotropic Fermi liquid, $1+2\rightarrow 3+4$ with $i\equiv(\mathrm{p}_i,\sigma_i)$, is given by 2π times the squared moduli of the quasi-particle scattering amplitude. We are interested in the situation that the momentum transfer $\mathrm{q}=\mathrm{p}_1-\mathrm{p}_3$ is small and $\mathrm{p}=\frac{1}{2}(\mathrm{p}_1+\mathrm{p}_3)$ and $\mathrm{p}'=\frac{1}{2}(\mathrm{p}_2+\mathrm{p}_4)$ are close to the Fermi momentum p_F. In this case, the scattering amplitude depends mainly on the relative orientation of the vector p, p$'$ and q, and on the the energy transfer $\omega=\epsilon_{\mathrm{p+q/2}}-\epsilon_{\mathrm{p-q/2}}=\epsilon_{\mathrm{p'+q/2}}-\epsilon_{\mathrm{p'-q/2}}$.

Quasi-particle scattering amplitude

We shall neglect the spin indices in the following for brevity. The spin-degeneracy factor 2 will be inserted when the need arises. The quasi-particle scattering amplitude $A_{\mathrm{pp'}}(\mathrm{q},\omega)$ is then given by the following equation,

$$A_{\mathrm{pp'}}(\mathrm{q},\omega)-\sum_{\mathrm{p''}}f_{\mathrm{pp''}}\chi_{0\mathrm{p''}}(\mathrm{q},\omega)\,A_{\mathrm{p''p'}}(\mathrm{q},\omega)=f_{\mathrm{pp'}},$$

where $\chi_{0\mathrm{p}}(\mathrm{q},\omega)$ is the susceptibility

$$\chi_{0\mathrm{p}}(\mathrm{q},\omega)=\frac{n^0_{\mathrm{p-q/2}}-n^0_{\mathrm{p+q/2}}}{\omega+\xi_{\mathrm{p-q/2}}-\xi_{\mathrm{p+q/2}}}\simeq\frac{\mathbf{q}\cdot\mathbf{v}_{\mathrm{p}}}{\mathrm{q}\cdot\mathrm{v}_{\mathrm{p}}-\omega}\frac{\partial n^0_{\mathrm{p}}}{\partial\xi_{\mathrm{p}}},$$

with $n^0_{\mathrm{k}}=n_F(\xi_{\mathrm{k}})$. We shall assume that the scattering is dominating by the $l=1$ channel and approximate

$$f_{\mathrm{pp'}}\,\tilde{}\,\frac{\mathbf{p}\cdot\mathbf{p}'}{p_F^2}f_1^s.$$

It is then easy to show that

$$A_{\mathrm{pp'}}(\mathrm{q},\omega)=\frac{\mathbf{p}\cdot\mathbf{p}'}{p_F^2}\frac{f_1^s}{1-\chi_1(\mathrm{q},\omega)f_1^s},$$

where

$$\chi_1(\mathrm{q},\omega)=\frac{1}{V}\sum_{\mathrm{p}}\frac{p^2}{p_F^2 d}\left(\frac{n^0_{\mathrm{p-q/2}}-n^0_{\mathrm{p+q/2}}}{\omega+\xi_{\mathrm{p-q/2}}-\xi_{\mathrm{p+q/2}}+i\delta}\right).$$

For small (q,ω) we have

$$\chi_1(\mathrm{q},\omega)\sim-\frac{N(0)}{d}\left[1+ig(d)\frac{\omega}{v_F q}\right]$$

for $\omega\ll v_F q$, where $q=|\mathrm{q}|$, $g(2)=1$ and $g(3)=\frac{\pi}{2}$. In the limit $N(0)f_1^s/d=F_1^s/d\rightarrow-1$, the scattering amplitude becomes

$$A_{\mathrm{pp'}}(\mathrm{q},\omega)\simeq\frac{\mathbf{p}\cdot\mathbf{p}'}{p_F^2}\frac{f_1^s}{1-\chi_1(\mathrm{q},\omega)f_1^s}.$$

Using the expansion

$$f_1^s=\frac{d}{N(0)}\left(-1-\beta\omega^2+\gamma_t q^2\right),$$

we obtain

$$A_{\mathrm{pp'}}(\mathrm{q},\omega)\simeq\frac{d}{N(0)}\frac{\mathbf{p}\cdot\mathbf{p}'}{p_F^2}\frac{1}{-ig\frac{\omega}{v_F q}+\gamma_t q^2},\tag{13}$$

where the last factor is exact the gauge field propagator in $U(1)$ gauge theory.

Thermal conductivity

Following Pethick[1], we shall use a variational approach[2] to derive the thermal conductivity κ from the transport equation. The thermal resistivity for a Fermi liquid is given by

$$\begin{aligned}\frac{1}{\kappa}&=\frac{1}{4}\sum_{1,2,3,4}W(1,2;3,4)n_1^0n_2^0(1-n_3^0)(1-n_4^0)\\&\times(\phi_1+\phi_2-\phi_3-\phi_4)^2\left(\sum_1\phi_1\xi_1\mathbf{v}_1\cdot\mathbf{u}\frac{\partial n_1^0}{\partial\epsilon_1}\right)^{-2}\\&\times\delta(\epsilon_1+\epsilon_2-\epsilon_3-\epsilon_4)\delta_{\sigma_1+\sigma_2,\sigma_3+\sigma_4}\delta_{\mathrm{p}_1+\mathrm{p}_2,\mathrm{p}_3+\mathrm{p}_4},\end{aligned}\tag{14}$$

where $n_i^0=n_F(\xi_i)$ is the Fermi distribution function with $i=1,2,3,4$, $\xi_i=\epsilon_i-\mu$, ϕ_i is defined by $n_i=n_i^0-\phi_i\frac{\partial n_i^0}{\partial\epsilon_i}$, v_i is the quasiparticle velocity, u is an arbitrary unit vector along which the temperature gradient ∇T is applied.

We shall take the usual trial function $\phi_i=\xi_i\mathbf{v}_i\cdot\mathbf{u}$. To the order we are working on the quasi-particle velocity may be replaced by v_F and

$$\begin{aligned}\sum_1\phi_1\xi_1\mathbf{v}_1\cdot\mathbf{u}\frac{\partial n_1^0}{\partial\epsilon_1}&=\sum_1(\xi_1\mathbf{v}_1\cdot\mathbf{u})^2\frac{\partial n_1^0}{\partial\epsilon_1}\\&=\frac{4m^*N(0)}{d}\int d\xi(\xi+\mu)\xi^2\frac{\partial n^0(\xi)}{\partial\xi}\\&=-\frac{4m^*N(0)}{d}\frac{\pi}{3}\epsilon_F(k_BT)^2\\&=-\frac{2\pi}{3}\frac{n}{m^*}(k_BT)^2,\end{aligned}$$

where n is the fermion density and the relation $d(n/m^*)=N(0)v_F^2$ is used.

Introducing $\bar{\xi}_{\mathrm{p}}=\frac{1}{2}(\xi_{\mathrm{p+q/2}}+\xi_{\mathrm{p-q/2}})$ and using the conditions $\mathrm{q}=\mathrm{p}_1-\mathrm{p}_3=\mathrm{p}_4-\mathrm{p}_2$ and $\omega=\epsilon_{\mathrm{p+q/2}}-\epsilon_{\mathrm{p-q/2}}=\epsilon_{\mathrm{p'+q/2}}-\epsilon_{\mathrm{p'-q/2}}$, we have

$$\begin{aligned}&m^*(\xi_1\mathbf{v}_1+\xi_2\mathbf{v}_2-\xi_3\mathbf{v}_3-\xi_4\mathbf{v}_4)\\=&\left(\bar{\xi}_{\mathrm{p}}+\omega/2\right)(\mathbf{p}+\mathbf{q}/2)+\left(\bar{\xi}_{\mathrm{p'}}-\omega/2\right)(\mathbf{p}'-\mathbf{q}/2)\\&-\left(\bar{\xi}_{\mathrm{p}}-\omega/2\right)(\mathbf{p}-\mathbf{q}/2)-\left(\bar{\xi}_{\mathrm{p'}}+\omega/2\right)(\mathbf{p}'+\mathbf{q}/2)\\=&\left(\bar{\xi}_{\mathrm{p}}-\bar{\xi}_{\mathrm{p'}}\right)\mathbf{q}+\omega(\mathbf{p}-\mathbf{p}')\\=&\left(\xi_{\mathrm{p}}-\xi_{\mathrm{p'}}\right)\mathbf{q}+\omega(\mathbf{p}-\mathbf{p}'),\end{aligned}$$

and

$$\begin{aligned}
&\langle(\phi_1+\phi_2-\phi_3-\phi_4)^2\rangle\\
&= \langle[(\xi_1\mathbf{v}_1+\xi_2\mathbf{v}_2-\xi_3\mathbf{v}_3-\xi_4\mathbf{v}_4)\cdot\mathbf{u}]^2\rangle\\
&= \frac{1}{d}(\xi_1\mathbf{v}_1+\xi_2\mathbf{v}_2-\xi_3\mathbf{v}_3-\xi_4\mathbf{v}_4)^2\\
&= \frac{1}{m^{*2}d}\left[(\xi_{\mathrm{p}}-\xi_{\mathrm{p}'})\mathbf{q}+(\mathbf{p}-\mathbf{p}')\omega\right]^2,
\end{aligned}$$

where $\langle\cdots\rangle$ means averaging over different u.

Putting $A_{\mathrm{pp}'}(\mathrm{q},\omega)$ into (14), and using the identity $n_1^0(1-n_3^0)=(n_1^0-n_3^0)/[1-e^{\beta(\epsilon_1-\epsilon_3)}]$, we obtain

$$\begin{aligned}
\frac{1}{\kappa} &\propto \frac{1}{T^4}\sum_{\mathrm{q,p,p'}}\int d\omega\frac{|A_{\mathrm{pp}'}(\mathrm{q},\omega)|^2}{(e^{\beta\omega}-1)(1-e^{-\beta\omega})}\\
&\times[n_F(\xi_{\mathrm{p-q/2}})-n_F(\xi_{\mathrm{p+q/2}})]\delta(\omega-\mathbf{p}\cdot\mathbf{q}/m^*)\\
&\times[n_F(\xi_{\mathrm{p'+q/2}})-n_F(\xi_{\mathrm{p'-q/2}})]\delta(\omega-\mathbf{p}'\cdot\mathbf{q}/m^*)\\
&\times[(\xi_{\mathrm{p}}-\xi_{\mathrm{p}'})^2q^2+(\mathbf{p}-\mathbf{p}')^2\omega^2],
\end{aligned}$$

where we have used the δ-functions to simplify the last line. Replacing $n_F(\xi_{\mathrm{p-q/2}})-n_F(\xi_{\mathrm{p+q/2}})$ and $n_F(\xi_{\mathrm{p'+q/2}})-n_F(\xi_{\mathrm{p'-q/2}})$ by $\omega\frac{\partial n_F}{\partial\xi_{\mathrm{p}}}$ and $\omega\frac{\partial n_F}{\partial\xi_{\mathrm{p}'}}$ respectively which is valid at small q and ω, we obtain

$$\begin{aligned}
\frac{1}{\kappa} &\propto \frac{1}{T^4}\sum_{\mathrm{q,p,p'}}\int d\omega\frac{\omega^2|A_{\mathrm{pp}'}(\mathrm{q},\omega)|^2}{(e^{\beta\omega}-1)(1-e^{-\beta\omega})}\\
&\times\frac{\partial n_F}{\partial\xi_{\mathrm{p}}}\frac{\partial n_F}{\partial\xi_{\mathrm{p}'}}\delta(\omega-\mathbf{p}\cdot\mathbf{q}/m^*)\delta(\omega-\mathbf{p}'\cdot\mathbf{q}/m^*)\\
&\times[(\xi_{\mathrm{p}}-\xi_{\mathrm{p}'})^2q^2+(\mathbf{p}-\mathbf{p}')^2\omega^2].
\end{aligned}$$

Let $\theta(\theta')$ be the angle between $\mathbf{p}(\mathbf{p}')$ and $\mathbf{q}$, and integrating over ξ_{p} and $\xi_{\mathrm{p}'}$, we obtain

$$\begin{aligned}
\frac{1}{\kappa} &\propto \frac{1}{T^4}\sum_{\mathrm{q},\hat{\mathrm{p}},\hat{\mathrm{p}}'}\int d\omega\frac{\omega^4|A_{\mathrm{pp}'}(\mathrm{q},\omega)|^2}{(e^{\beta\omega}-1)(1-e^{-\beta\omega})}(\hat{\mathbf{p}}-\hat{\mathbf{p}}')^2\\
&\times\delta(\omega-qv_F\cos\theta)\delta(\omega-qv_F\cos\theta').
\end{aligned}$$

Assuming that the scattering is dominated by F_1^s channel and using Eq.(13), we obtain

$$\begin{aligned}
\frac{1}{\kappa} &\propto \frac{1}{T^4}\sum_{\mathrm{q}}\int d\omega\frac{\omega^4}{(e^{\beta\omega}-1)(1-e^{-\beta\omega})}\frac{1}{g^2\frac{\omega^2}{v_F^2q^2}+\gamma_t^2q^4}\\
&\propto \frac{1}{T^4}\int d\omega\frac{\omega^4}{(e^{\beta\omega}-1)(1-e^{-\beta\omega})}\omega^{-(4-d)/3}\\
&\propto \left(\frac{k_BT}{\epsilon_F}\right)^{(d-1)/3}.
\end{aligned}$$

which is the thermal resistivity coming from inelastic scattering between fermions. At low temperature the inelastic scattering is cut off by elastic impurity scattering rate $1/\tau_0$ which gives rise to

$$\frac{\kappa_{\mathrm{el}}}{T}=\frac{1}{d}\gamma^*v_F^2\tau_0,$$

where $\gamma^*=C_V/T$ is the specific heat ratio. The total thermal conductivity is therefore given by

$$\frac{\kappa}{T}\propto\max\left[\frac{\hbar}{k_B^2}\left(\frac{k_BT}{\epsilon_F}\right)^{(4-d)/3},\frac{d}{\gamma^*v_F^2}\frac{1}{\tau_0}\right]^{-1}.$$

[1] C. J. Pethick, Phys. Rev. 177 (1969).

[2] J. M. Ziman, *Electrons and Phonons*, Oxford University Press, New York, (1960).